

THE TERRAN SERIES

Document XIV of XIV

The Terran Sphere

The Geometry of Earthly Time

***Coherence written into time must first be written into space —
and the space is a sphere.***

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Preamble

Document I of the Terran Series opened with a proposition:

Life is coherence that has learned to write itself into time.

That proposition established what time is — the medium through which living systems generate and sustain coherence. It grounded thirteen documents of calendar, zodiac, practice, and instrument.

Document XIV begins one step before. Before coherence can write itself into time, time must live somewhere. And the somewhere is not abstract. It is specific, geometrically precise, and consequential for every human being on Earth.

The Earth is a sphere. It is tilted at 23.436 degrees relative to its orbital plane. It rotates once every 23 hours, 56 minutes, and 4 seconds. It orbits the sun in 365.2422 days along an elliptical path that brings it closest to the sun in early January and furthest in early July.

These are not background facts. They are the primary facts — the ones every honest timekeeping system must answer to. Every calendar that ignores them generates disorder. Every calendar that honors them generates coherence.

This document shows the geometry. It shows what a sphere tilted at 23.436 degrees does to the experience of time at different latitudes. It shows why the same calendar date means fundamentally different things in Oslo and Sydney. It shows how spherical trigonometry — the mathematics of the sphere — is the connective tissue between the physical foundation of time, the organizational structure of the Terran Calendar, and the meaning layer that makes time livable rather than merely measurable.

Coherence written into time must first be written into space — and the space is a sphere.

Part I — The Sphere

I.1 The Primary Facts

Three numbers determine the temporal experience of every human being on Earth:

23.436° — *the obliquity of the ecliptic. The angle at which Earth's rotational axis tilts relative to its orbital plane. This single angle is the origin of all seasons. Without it, every day on Earth would be 12 hours long everywhere, every month would be climatically identical, and the concept of a seasonal calendar would be meaningless.*

0.0167 — *Earth's orbital eccentricity. The degree to which Earth's orbit deviates from a perfect circle. This number determines that Earth reaches perihelion — its closest point to the sun — in early January, and aphelion — its furthest point — in early July. It is the origin of the 6.9% solar flux asymmetry between the hemispheres.*

φ — *the observer's latitude. The angular distance from the equator. This number determines what the obliquity and eccentricity actually mean for a specific human body at a specific location on the sphere. Two people experiencing the same calendar date may be experiencing opposite seasons, opposite day lengths, and opposite relationships to the sun — because they stand at different values of φ .*

These three numbers — obliquity, eccentricity, latitude — are the primary facts of earthly time. Every honest timekeeping system is an answer to all three simultaneously. Every dishonest one ignores at least one of them.

I.2 The Obliquity of the Ecliptic

The ecliptic is the plane of Earth's orbit around the sun. The celestial equator is the projection of Earth's equatorial plane onto the celestial sphere. The angle between them — 23.436 degrees — is the obliquity of the ecliptic.

This angle is not static. It oscillates between approximately 22.1 and 24.5 degrees over a 41,000-year cycle driven by gravitational interactions with other planets. At present it is

23.436 degrees and slowly decreasing. But for all practical purposes of calendar design — across centuries of human civilization — it is the fixed angle that makes seasons possible.

The obliquity means that as Earth orbits the sun, the Northern Hemisphere tilts toward the sun for half the year (NH summer) and away for the other half (NH winter). The Southern Hemisphere does the opposite — simultaneously. This simultaneity is the physical foundation of everything the Terran Series calls hemispheric asymmetry. It is not a philosophical position. It is geometry.

I.3 Orbital Eccentricity and the Flux Asymmetry

Earth's orbit is not a circle. It is an ellipse — slightly, but consequentially. The eccentricity of 0.0167 means that Earth's distance from the sun varies between 147.1 million kilometers at perihelion (January 3-5) and 152.1 million kilometers at aphelion (July 4-6).

By the inverse square law, solar flux — the amount of solar energy reaching Earth — varies with the square of distance. The ratio of aphelion to perihelion distance, squared, gives the flux ratio:

$$\text{Flux ratio} = (152.1 / 147.1)^2 = 1.069$$

This means the Southern Hemisphere's summer receives approximately 6.9% more solar energy than the Northern Hemisphere's summer. This is not a small difference. It is the equivalent of moving several hundred kilometers closer to the equator. It contributes directly to the asymmetry in climate patterns, seasonal amplitudes, and biological rhythms between the hemispheres.

The Gregorian calendar treats this asymmetry as irrelevant. The Terran Series treats it as a primary fact. Document X (The Terran Meridian) documents the physical evidence. This document provides the geometric proof.

I.4 The Latitude Problem

The most consequential number in any individual's temporal experience is their latitude — the angular distance from the equator, measured in degrees north or south.

Latitude determines day length. Day length is calculated using the fundamental formula of spherical trigonometry:

$$\cos(H) = -\tan(\varphi) \times \tan(\delta)$$

Where H is the hour angle of sunrise and sunset, φ is the observer's latitude, and δ is the solar declination — the angle between the sun's position and the celestial equator, which varies between $+23.436^\circ$ at the June solstice and -23.436° at the December solstice.

Day length in hours is then:

$$\text{Day length} = 2H / 15$$

This formula is the mathematical proof that the same calendar date produces fundamentally different solar experience at different latitudes. The table below makes that proof visible:

Latitude	City	June Solstice	Equinox	Dec Solstice
+60°	Oslo	18.49 h	12.00 h	5.51 h
+40°	New York	14.84 h	12.00 h	9.16 h
+23.5°	Tropic of Cancer	13.45 h	12.00 h	10.55 h
0°	Equator	12.00 h	12.00 h	12.00 h
-23.5°	Tropic of Capricorn	10.55 h	12.00 h	13.45 h
-34°	Sydney	9.73 h	12.00 h	14.27 h
-60°	Antarctic Circle	5.51 h	12.00 h	18.49 h

Table 1: Day length at key latitudes for three representative dates. Calculated using $\cos(H) = -\tan(\varphi) \times \tan(\delta)$. All values in hours.

The numbers are unambiguous. On June 21, a person in Oslo experiences 18.49 hours of daylight. A person in Sydney experiences 9.73 hours. They share the same calendar date. They do not share the same temporal experience. A calendar that assigns them identical meaning for that date is not just culturally insensitive — it is geometrically wrong.

The Terran Calendar does not make that error. By anchoring the Northern Hemisphere year to April 1 (spring) and the Southern Hemisphere year to October 15 (spring), the Terran Calendar honors what the formula above proves: that seasonal experience is a function of latitude, and that a planetary calendar must account for both poles of the sphere simultaneously.

Part II — The Ecliptic and the Thirteen

II.1 The Ecliptic Coordinate System

The ecliptic is not only the plane of Earth's orbit — it is the reference plane for measuring the positions of the zodiac constellations. Ecliptic longitude measures position along the ecliptic from 0° (the vernal equinox point) to 360° . Ecliptic latitude measures angular distance above or below the ecliptic plane.

The traditional twelve zodiac constellations were defined by their ecliptic longitude ranges — each covering approximately 30° of the 360° ecliptic circle. This is the origin of the twelve-sign zodiac: 360° divided by 12 equals 30° per sign, a clean result that appealed to ancient mathematical sensibility.

But the actual boundaries of the constellations along the ecliptic are not equal. The International Astronomical Union (IAU) defines constellation boundaries that give the sun very different amounts of time in each constellation. The sun spends approximately 45 days in Virgo, 38 days in Pisces, and only 7 days in Scorpius — before crossing into Ophiuchus for 18 days.

This is the astronomical reality that the Terran Series honors. The traditional twelve-sign zodiac imposes administrative equality (30° each) on a geometrically unequal sky. The Terran thirteen-sign zodiac does not claim equal time in each constellation — it claims honest structural correspondence between the 13 Terran months and the 13 signs, with Ophiuchus and Zola sharing the same structural position as the thirteenth.

II.2 Ophiuchus in Ecliptic Coordinates

Ophiuchus — the Serpent Bearer — occupies ecliptic longitudes approximately 237° to 271° . The sun crosses Ophiuchus from approximately November 29 to December 17 each year. This is the astronomical fact that leads some to say "Ophiuchus is in November" — and they are correct in astronomical terms.

The Terran Series places Ophiuchus at Zola (September 16 – October 14) for a different reason: structural correspondence, not astronomical position tracking. The Terran Calendar maps 13 signs to 13 months by position in the organizational structure, the same way the tropical zodiac maps 12 signs to 12 months by position from the vernal equinox — not by tracking the sun's actual constellation.

Both systems make organizational choices. The Terran Series makes its choice explicitly and explains it. Zola is the 13th month. Ophiuchus is the 13th sign. They belong together not because of where the sun is on November 29 but because they are the same structural truth expressed in two different systems — the calendar and the sky.

II.3 The {13/5} Star on the Celestial Sphere

The {13/5} star — the geometric heart of the Terran Soul Star (Document II) — is a two-dimensional projection of relationships that exist on the celestial sphere. Each of its 13 points corresponds to a zodiac sign. Each sign occupies a position on the ecliptic. The geometry of the star — coprime 13 and 5, a single unbroken path, the interior lattice of triangles and diamonds — reflects the organizational coherence of 13 signs distributed around a 360° circle.

The angular separation between adjacent points of a {13/5} star is $5 \times (360^\circ/13) = 138.46^\circ$. This angle appears in the actual ecliptic spacing of the zodiac constellations when organized by the Terran system. It is not arbitrary decoration. It is the geometry of a 13-fold division of the ecliptic expressed as a star polygon.

The Northern Hemisphere perceives the triangular points of the {13/5} star — the sharp outward projections of the organizational structure. The Southern Hemisphere perceives the diamond shapes — the interior rhombuses formed by the crossing lines. Both geometric families inhabit the same unbroken path. Ophiuchus at the center belongs to

neither hemisphere — it is the convergence of both. This geometric description is not metaphorical. It corresponds to the actual asymmetric relationship between NH and SH seasonal experience as a function of latitude and obliquity.

Part III — The Geometry of Containment

III.1 The Soul Star and the Old System

The question every reader eventually arrives at is this: what happens to the old system? When the Terran Calendar proposes thirteen months where twelve have stood for centuries, it must answer whether the old structure is displaced — overwritten, discarded, made irrelevant. The answer, stated plainly, is no. The old system is not displaced. It is contained.

This is not a philosophical concession. It is a geometric fact. A regular 13-gon inscribed in a circle geometrically contains a regular 12-gon inscribed in the same circle. The 13-gon is denser — its vertices fall every 27.69° around the circle, against the 12-gon's 30° . When the {13/5} Soul Star is drawn on those 13 points — connecting every 5th point in a single unbroken path — its web of crossing lines encloses the 12-gon's space entirely. The 12-gon is still there, inscribed in the same sphere. It has not been erased. It has been surrounded.

The dodecahedron — the Platonic solid whose twelve pentagonal faces have made it a symbol of cosmic completeness across cultures and centuries — encodes the same 12-fold symmetry in three dimensions. It too can be inscribed in a sphere. Earth, at the center of that sphere, is surrounded by both the dodecahedron's twelve faces and, extending beyond them, the web of the {13/5} Soul Star drawn on the same circumscribing circle. The geometry is consistent across dimensions: the 12-fold structure occupies the interior, and the 13-fold structure contains it.

III.2 The Shared Step of Five

The containment relationship between the two systems is not arbitrary. It is reinforced by a structural bridge: both the {13/5} Soul Star and the dodecahedron are generated by the number five.

The Soul Star is drawn by connecting every 5th of its 13 points. The step interval — 5 — is what creates the star's characteristic crossing pattern, its interior lattice of triangles and diamonds, and the single unbroken path that visits all 13 points before returning to its origin. Remove the step of 5 and the star ceases to exist. The number 5 is not incidental to the $\{13/5\}$ star; it is definitional.

The dodecahedron is built entirely from pentagons — twelve regular pentagons, each with five sides, joined at their edges to form a closed solid. The pentagon carries the golden ratio in its structure: the ratio of a pentagon's diagonal to its side is exactly $\phi = (1 + \sqrt{5}) / 2$. The dodecahedron therefore encodes ϕ at every face. The $\{13/5\}$ Soul Star, through its step-5 chord ratios, carries internal proportions that approach ϕ — not identically, but with genuine geometric kinship. Both structures are expressions of five-fold symmetry, one in two dimensions on the ecliptic circle, the other in three dimensions as a Platonic solid.

Three statements follow from this geometry, each more precise than the last:

A regular 13-gon drawn on the same circle as a regular 12-gon geometrically contains it — the $\{13/5\}$ Soul Star's web encloses the 12-fold space.

Both structures share the number 5 as their generative step: the Soul Star connects every 5th of its 13 points; the dodecahedron is built entirely from pentagons.

The Terran Calendar does not displace the Gregorian system — it contains it, as the 13-gon contains the 12-gon inscribed in the same sphere.

This is the geometric character of the Terran reform. It does not ask the old system to disappear. It builds a structure that is larger — more honest, more complete, more aligned with the sphere — and the old structure fits inside it. Anyone who needs to translate between the Terran Calendar and the Gregorian calendar can do so. The 12 is still there, inscribed in the same circle. The 13 simply reaches further.

Part IV — The Connective Tissue

IV.1 Three Layers, One Geometry

The Terran Series has always operated across three layers simultaneously — the astronomical foundation, the organizational structure, and the meaning layer. Until this document, the connection between these layers has been described philosophically. Spherical trigonometry makes that connection mathematical.

The infrastructure layer — the physical reality of time — is expressed in the geometry of the sphere: obliquity, eccentricity, latitude. These are the inputs to the spherical trigonometry formulae that calculate day length, solar altitude, and solar flux at any point on Earth at any moment in the year.

The organization layer — the Terran Calendar — is expressed in the structural choices that honor that geometry: two hemispheric anchors at opposite seasons, 13 months of 28 days that correspond to 13 organizational positions on the ecliptic, Zola as the planetary pause absorbing the orbital remainder that no clean structure can contain.

The meaning layer — the soul calendar, the practice, the weekly questions — is expressed in the biological and psychological consequences of the geometry: the way day length affects circadian rhythms, the way solar flux shapes seasonal mood and energy, the way the body knows what time of year it is before the mind consults a calendar.

Spherical trigonometry is the language in which all three layers speak to each other. The formula $\cos(H) = -\tan(\varphi) \times \tan(\delta)$ connects the physics of the obliquity (δ) to the experience of the observer (φ) to the organizational structure of the calendar (the seasonal anchors that correspond to specific values of δ). It is one formula that spans all three layers.

IV.2 The Entropy of Ignoring the Sphere

The Universal Times framework (Gossett, 2026) defines temporal entropy as time coordination, sequencing, latency, scheduling, and settlement disorder. It proposes that value is generated by reducing entropy without hiding the cost — exporting disorder to another domain.

The Gregorian calendar generates temporal entropy in precisely the way this framework identifies as illegitimate: it reduces administrative complexity for the Northern Hemisphere by exporting calendrical disorder to the Southern Hemisphere. The cost is hidden because the people who designed the system were not the people who absorbed the disorder.

This is geometrically demonstrable. At latitude -34° (Sydney), the Gregorian calendar assigns January as the first month of the year — mid-summer in the Southern Hemisphere. New Year falls in the middle of the hottest, most energetically peak period of the SH annual cycle, creating a fundamental misalignment between administrative time and biological time for the 2 billion people who live south of the equator.

The Terran Calendar eliminates this disorder by anchoring both hemispheres to their respective spring equinoxes: April 1 for NH, October 15 for SH. The entropy is not redistributed — it is genuinely reduced. The calendar aligns with the geometry of the sphere rather than the administrative convenience of one hemisphere's dominant civilization.

Spherical trigonometry provides the proof. The day length formula shows precisely how much the solar experience diverges between hemispheres on any given date. The flux calculation shows precisely how much more solar energy the SH receives in its summer. These are not soft cultural claims. They are measurable, calculable, geometrically exact quantities that the Gregorian calendar ignores and the Terran Calendar honors.

IV.3 The Biological Bridge

The meaning layer of the Terran Series — the soul months, the movement names, the weekly practice questions — is not arbitrary. It is grounded in the biological consequences of the geometry described in this document.

Day length affects the production of melatonin and serotonin, the regulation of cortisol, the timing of the sleep-wake cycle. Solar flux affects vitamin D synthesis, immune function, and mood regulation. The progression from short days to long days and back — the annual rhythm of light — is the primary environmental signal that biological clocks use to orient the body in time.

The soul months of the Terran Calendar correspond to the biological reality of that annual rhythm — differently for the Northern and Southern Hemispheres, because the formula $\cos(H) = -\tan(\varphi) \times \tan(\delta)$ produces different day lengths for different latitudes. Acerto in NH spring corresponds to lengthening days, rising solar altitude, and the biological awakening that follows the winter minimum. Acerto in SH spring corresponds to the same biological process — but shifted 197 days, because the geometry of the sphere demands it.

The neuroscience of temporal experience — the third layer of the project initiated by this document — will provide the biological mechanism that connects the geometry to the experience. Spherical trigonometry shows what the sphere does. Neuroscience will show what the body does with that information. The soul calendar names what it means.

IV.4 The First Derivative

The spherical trigonometry of day length gives us $L(d, \varphi) = (24/\pi) \times \arccos(-\tan(\varphi) \times \tan(\delta(d)))$, where $\delta(d)$ is the solar declination as a function of day of year. This is the formula at the heart of the Terran Calendar's geometric foundation. But the formula describes a position — how long the day is on any given day at any latitude. The more revealing quantity is the rate at which that position is changing. That requires the first derivative.

By the chain rule, dL/dd — the rate of change of day length with respect to day of year — is:

$$dL/dd = (24/\pi) \times [\tan(\varphi) \times \sec^2(\delta) / \sqrt{1 - \tan^2(\varphi) \times \tan^2(\delta)}] \times (2\pi \times 0.40928 / 365) \times \sin(2\pi(d+10)/365)$$

This derivative has a precise and remarkable behaviour across the year. At both equinoxes — March ($d \approx 80$) and September ($d \approx 266$) — dL/dd reaches its maximum value. At both solstices — June ($d \approx 172$) and December ($d \approx 355$) — dL/dd falls to approximately zero. This is not a curiosity. It is the mathematical origin of the word solstice itself: sol (sun) + sistere (to stand still). The sun does not literally stop. The rate of change of its light reaches zero. The derivative vanishes.

The magnitude of dL/dd is latitude-dependent. At $45^\circ N$ — the latitude of cities like Lyon, Minneapolis, and Vladivostok — day length changes at approximately 3.2 minutes per day at the equinox. At $60^\circ N$ — Helsinki, Anchorage, St. Petersburg — it reaches nearly 5.6 minutes per day. The body is receiving a measurably stronger rate-of-change signal at higher latitudes, which corresponds precisely to the more pronounced seasonal biology observed in those populations.

The Terran Calendar anchors both hemispheres at the moments of maximum dL/dd . The Northern Hemisphere anchor is the March equinox — the moment when dL/dd peaks in the north. The Southern Hemisphere anchor is the September equinox — the moment when dL/dd peaks in the south. These are not philosophical preferences or cultural inheritances. They are the mathematically optimal moments to mark the year's turning — the points at which the biological signal of changing light is at its strongest.

The Gregorian calendar anchors to January 1. January 1 falls near the December solstice, where $dL/dd \approx 0$ — the moment of minimum biological signal, the point at which the rate of change of daylight is nearest to zero. The new year begins precisely when the solar signal is least informative. The Terran Calendar begins precisely when it is most informative. That is the calculus argument for the equinox anchor.

IV.5 Kinesis — The Clock of Light

Kinesis is the first instrument to express a person's position within today's photoperiod as a decimal fraction of the available daylight, grounded in the spherical geometry that determines that photoperiod.

The tradition it belongs to is ancient. The Greeks and Romans used temporal hours — twelve equal divisions of the daylight period, each longer in summer and shorter in winter. Medieval monasteries used canonical hours from sunrise to sunset for the same reason. Both systems honored a truth that the modern clock abandoned: that the meaningful unit of daily time is not a fixed administrative interval but a fraction of the light the sphere delivers to a body at a given latitude on a given day. Both systems were eventually

abandoned because their variable units were inconvenient for coordination across distances. Kinesis does not abandon the insight. It completes it.

What is new in Kinesis is not the variable light-hour — that is two thousand years old. What is new is the combination of three things. First: decimal precision grounded in spherical trigonometry. The duration of one Kinesis Pulse is derived from $\cos(H) = -\tan(\phi) \times \tan(\delta)$ — the same formula that determines day length, solar altitude, and solar flux at any latitude on any day of the year. The ancient temporal hour was observed; the Kinesis Pulse is calculated.

Second: the breathing pulse as biological signal. The rate at which the Kinesis Pulse changes duration across the year is precisely the Kairos signal — dL/dd , the first derivative of day length. Near the equinoxes, when the biological signal of seasonal change is strongest, the pulse duration is changing fastest. Near the solstices, when the sun stands still and the signal stabilizes, the pulse duration changes slowest. The year's turning is felt in the rhythm of the clock.

Third: the honest boundary. Kinesis stops at sunset and restarts at sunrise. Every other clock runs through the night as if nothing has changed. Kinesis does not. The night is not a continuation of the day. A clock that counts light should count only light.

The claim is precise: Kinesis is the first live instrument to measure position within the photoperiod as a decimal fraction derived from spherical trigonometry, with a pulse duration that encodes the annual biological light signal. It is available at terranseries.com as Instrument VI of the Terran Sphere.

IV.6 La Curva di Viviani — The Shape of the Infinity Clock

In 1659, the Italian mathematician Vincenzo Viviani — student and biographer of Galileo — described a remarkable curve: the intersection of a sphere with a cylinder that passes through the center of the sphere and is tangent to it at one point. The result is a closed figure-eight curve that lies entirely on the surface of a sphere, crosses itself at exactly one point, and makes two complete loops before returning to its starting position.

The Terran Infinity Clock is this curve. Not metaphorically. Geometrically.

The Parametric Form

The Viviani curve is defined by:

$$x(t) = a(1 + \cos t)$$

$$y(t) = a \sin t$$

$$z(t) = 2a \sin(t/2)$$

where t runs from 0 to 4π for one complete traversal and a is a scaling constant. The curve lies on the sphere of radius $2a$, verified by the identity:

$$x^2 + y^2 + z^2 = 4a^2 \text{ (for all } t\text{)}$$

The Mapping to the Infinity Clock

Every feature of the Viviani curve maps exactly to the Infinity Clock:

- The crossing point ($t = 0, t = 4\pi$): $x = 2a, y = 0, z = 0$. This is Zola $\mathfrak{Z}$ — the one moment in the Terran Year when both hemispheres meet simultaneously. The Viviani curve crosses itself here and only here. The crossing is not arbitrary — it is the point where the curve is tangent to the equatorial plane ($z = 0$). Zola is the equatorial moment of the Terran Year.
- The Northern Hemisphere ring: $t \in [0, 2\pi], z \geq 0$. The upper loop of the Viviani curve. Maximum $z = 2a$ at $t = \pi$, corresponding to the NH September equinox — the midpoint of the NH year. The loop ascends from Zola, reaches its highest point at the September equinox, and returns to Zola.
- The Southern Hemisphere ring: $t \in [2\pi, 4\pi], z \leq 0$. The lower loop of the Viviani curve. Minimum $z = -2a$ at $t = 3\pi$, corresponding to the SH March equinox — the midpoint of the SH year. The loop descends from Zola, reaches its deepest point at the March equinox, and returns to Zola.
- Full closure: after 4π of parametric travel — two complete loops, one for each hemisphere — the curve returns to its starting point. One Terran Year, two hemispheric experiences, one geometric object. $\gamma(0) = \gamma(4\pi)$. The Terran Cycle closes.

The Sphere

Setting $a = 6$ months (half the Terran Year), the Viviani curve lies on a sphere of radius $2a = 12$ months — one complete year. The sphere is the Terran Year as a geometric object. The Viviani curve is the path that hemisphere-honest timekeeping traces on the surface of that sphere.

The Viviani curve is the only closed curve on a sphere that makes exactly two loops and crosses itself exactly once. There is no other curve with this topology. The Infinity Clock's figure-eight is not one of many possible shapes — it is the unique shape that the hemisphere-honest Terran Year requires. The geometry chose it. The Series built it. Viviani named it.

Vincenzo Viviani and the Gregorian Calendar

Vincenzo Viviani discovered this curve in 1659. The Gregorian calendar — the calendar the Terran Series proposes to reform — was consolidated in 1582, seventy-seven years before Viviani wrote down the equations of the curve that describes hemisphere-honest timekeeping. The curve waited 364 years for the Terran Series to place the Terran Year on its surface and name the crossing point Zola.

IV.7 Gravitropism — The Biological Viviani

There is a biological analog to the Viviani curve that has been present in every living thing on Earth since the first organisms rooted themselves in the ground: gravitropism. Gravitropism is the orientation of living organisms in response to gravity — roots growing downward toward Earth's center, shoots growing upward away from it. It is the most elementary biological response to the sphere — the body's answer to the same gravitational geometry that defines every Terran Position.

The mechanism is precise: gravity causes asymmetric distribution of auxin, a plant growth hormone, which causes differential cell elongation on opposite sides of the root or shoot. The result is directional growth — always relative to the sphere's center, always oriented along the gravitational axis that every point on Earth shares.

The Viviani curve's z-axis is this gravitational axis. The Northern Hemisphere loop rises above $z = 0$ (upward from the equatorial plane). The Southern Hemisphere loop descends below $z = 0$ (downward). The crossing point at Zola sits exactly at $z = 0$ — the equatorial plane where the gravitational axis is horizontal, where neither hemisphere is above or below the other, where the sphere treats both sides identically. This is not coincidence. The Viviani curve lives on a sphere. Gravitropism orients to a sphere. They share the same geometry because they share the same object.

Viviani posed his curve as an architectural problem: how do you illuminate a hemispherical dome with four equal windows? The answer required cutting the dome with cylindrical shafts whose intersection curves are Viviani curves. Light entering a hemispherical dome. The same geometry as a planet illuminated by a star, with the terminator line — already drawn by the Sphere Clock in real time — as the boundary between light and dark. Gravitropism orients to the sphere's gravity. Phototropism orients to the sun's light. The Viviani curve encodes both: its z-axis is the gravitational axis, its traversal is driven by the annual light cycle.

The two biological orientations of every living thing on Earth are gravity and light. The two geometric features of the Viviani curve are its spherical surface (gravity) and its traversal path (light). The Terran Series has always argued that biological time and geometric time are the same thing seen from two different angles. Gravitropism and the Viviani curve are the same thing seen from two different kingdoms — one biological, one mathematical.

The Viviani curve is what the sphere's geometry requires. Gravitropism is what the sphere's gravity produces. Both shape the living world in the same figure-eight pattern — one in time, one in space. The Infinity Clock traces the curve in time. Every plant on Earth traces it in growth.

IV.8 The Two Hinges — Zola and Kashf

The Terran Year is not a straight sequence of thirteen months. It is a folded structure with two hinge points — two moments where the year turns, bends, or crosses itself. The

seasonal hinge carries an Arabic name: Kashf (كشف, revelation). Zola 𐤆 is Series-named — the crossing point named for hemispheric simultaneity. And the full hinge zone — months 9, 10, and 11 — is entirely Arabic: Itiwā' (light), Kashf (revelation), Taḥawwul (transformation). Arabic gave the Series the language of the seasonal turn.

The complete architecture of the 4+5+4 Movement structure:

- Dissolution (months 1–4): Acerto, Encarar, Tebanasu, Msingi — the year opening, light releasing
- Recognition (months 5–6): Common, Daichi — ascending toward the cosmic hinge
- Zola 𐤆 (month 7): THE COSMIC HINGE — both hemispheres meet, the Viviani curve crosses itself, $z = 0$
- Recognition (months 8–9): Tecido, Itiwā' — descending from the cosmic hinge
- Kashf (month 10): THE SEASONAL HINGE — revelation, the turn from expression to consolidation
- Inscription (months 11–13): Taḥawwul, Kuumba, Chuánchéng — the year closing, light gathering

Zola 𐤆 — The Cosmic Hinge

Zola is month 7 — the exact center of Recognition and the exact center of the thirteen-month year. It is the equinoctial crossing: the moment when both hemispheres share the same seasonal position simultaneously. In the geometry of La Curva di Viviani, Zola is the crossing point ($z = 0$, $x = 2a$, $y = 0$) where the Northern Hemisphere loop and the Southern Hemisphere loop meet. The NH loop ascends from Zola, reaches its maximum, and returns. The SH loop descends from Zola, reaches its minimum, and returns. The entire figure-eight is organized around this crossing.

Zola is the hinge around which the year folds. Before Zola, the year is ascending toward the crossing. After Zola, the year is descending from it. The hinge requires Recognition to be an odd number of months — 5 — so that Zola can sit at the exact center: 2 months before, Zola, 2 months after.

Kashf — The Seasonal Hinge

Kashf (Arabic: كشف — revelation, unveiling, the bringing to light of what was hidden) is month 10 — the first month of Inscription and the boundary between Recognition and Inscription. It is not simply the start of a new Movement. It is the turn: the moment the year moves from expression toward consolidation, from speaking toward writing.

Recognition expresses. Inscription records. Kashf is the threshold between them — the unveiling that precedes the writing. What was shown through Recognition is now ready to be inscribed. Kashf names that threshold precisely: revelation is what happens when the hidden becomes visible, when the year's meaning surfaces before it is committed to the record.

That Taḥawwul (transformation) follows Kashf is the Series' most precise sequential logic: the revelation (Kashf) precedes the transformation (Taḥawwul), which precedes the creativity (Kuumba), which precedes the inheritance (Chuánchéng). The Inscription Movement is a four-step process: see it clearly, be changed by it, make something from it, pass it forward.

IV.9 The Arabic Hinge Zone

The seasonal hinge carries an Arabic name — Kashf (كشف, revelation). Zola ٩ is Series-named, not Arabic: it is the crossing point named for hemispheric simultaneity. Kashf is the seasonal hinge — the turn from expression to consolidation, the boundary between Recognition and Inscription. And three consecutive months span the hinge zone entirely in Arabic: Iṭiwā' (light, month 9), Kashf (revelation, month 10), Taḥawwul (transformation, month 11). The year does not turn at a single point. It moves through a zone — three months, one language, one arc of meaning. The language of the seasonal turn is Arabic.

The year folds at Zola. It turns at Kashf.

Part V — The Entropy Accounting

The case for the Terran Calendar is not merely philosophical. It is geometrically auditable. The following accounting uses the framework proposed in *Universal Times* (Gossett, 2026) to demonstrate that the Terran Calendar reduces temporal entropy without exporting disorder elsewhere.

Claim: *The Gregorian calendar generates measurable temporal entropy at the Southern Hemisphere and equatorial latitudes. The Terran Calendar reduces this entropy by anchoring both hemispheres to geometrically honest seasonal positions.*

Evidence 1 — Day length misalignment. *The formula $\cos(H) = -\tan(\varphi) \times \tan(\delta)$ shows that January 1 (Gregorian New Year) occurs at solar declination $\delta = -23^\circ$. At latitude $\varphi = -34^\circ$ (Sydney), this produces a day length of 14.27 hours — peak summer. At $\varphi = +52^\circ$ (London), this produces a day length of 7.99 hours — deep winter. The same administrative date corresponds to opposite biological seasons. This is not disorder in a metaphorical sense. It is measurable misalignment between social time and biological time.*

Evidence 2 — Flux asymmetry ignored. *The 6.9% perihelion flux advantage of the SH summer is real and calculable. The Gregorian calendar contains no acknowledgment of this asymmetry. It treats all latitudes as equally affected by the same calendar structure. The Terran Calendar does not claim to change the physics — but it organizes time around hemispheric reality rather than ignoring it.*

Evidence 3 — Remainder handling. *The Gregorian calendar distributes the orbital remainder across multiple months of unequal length — 28, 29, 30, or 31 days depending on month and year. This creates scheduling irregularity that is itself a form of temporal entropy: different months contain different numbers of working days, weeks do not align with month boundaries, and payroll and planning cycles require constant correction. The Terran Calendar quarantines the remainder in Foundation Day and Leap Day — outside all week-counting — so that Months 1 through 13 are perfectly regular.*

Verdict: *The Terran Calendar reduces temporal entropy at the structural layer (28-day months, clean remainder), at the hemispheric layer (dual seasonal anchors), and*

at the meaning layer (soul months aligned to biological rhythms). It does not export this disorder elsewhere. The entropy is genuinely reduced.

Part VI — What the Sphere Completes

Document I gave the Terran Series its philosophical foundation: life is coherence that has learned to write itself into time. That proposition established the what and the why of the Series.

Document XIV gives it the geometric foundation: coherence written into time must first be written into space — and the space is a sphere. That proposition establishes the where — the specific, measurable, latitude-dependent space that every honest account of earthly time must honor.

Together, these two propositions form a complete philosophical arc:

Life is coherence that has learned to write itself into time.

***Coherence written into time must first be written into space — and
the space is a sphere.***

The first tells you what time is. The second tells you where time lives. The thirteen documents between them show you how to inhabit that time honestly — as a living being at a specific latitude, in a specific hemisphere, on a specific planet whose geometry shapes every moment of your biological existence.

Spherical trigonometry is not an addition to the Terran Series. It is the mathematical confirmation that the Series was always right — that hemispheric honesty is not a philosophical preference but a geometric necessity, that the 28-day month is not arbitrary but biologically grounded, that Zola belongs at the threshold because the orbital remainder is a geometric fact, and that the soul calendar names what the sphere does to the body.

The three-layer project — infrastructure, organization, and meaning — now has its connective tissue. The hydrogen hyperfine transition grounds time in physics. Spherical

trigonometry connects that physics to the specific planet and the specific observer. The Terran Calendar organizes that connection into a livable structure. The soul calendar names what that structure means.

The neuroscience of temporal experience will complete the third layer — showing what the body does with the light the sphere provides, and why the naming matters neurologically and not just philosophically. That work remains to be done. This document prepares the ground.

The Remaining Six Triangles

Triangle 1 — the Astronomical Triangle — is the foundation. But the Terran Series uses six more spherical triangles, each producing a different instrument, each capturing a different relationship on the sphere. Together the seven form a complete geometric family.

Triangle 2 — The Declination Triangle (E-S-V)

Before the Astronomical Triangle can be solved, the solar declination δ must be known. δ is itself the product of a spherical triangle — the Declination Triangle connecting the Sun (S), the Vernal Equinox (V), and the point where the sun's meridian meets the celestial equator (E).

The three vertices:

- S — the Sun, at its position on the ecliptic
- V — the Vernal Equinox, where the ecliptic crosses the celestial equator
- E — the foot of the perpendicular from S to the celestial equator

The key angle is the obliquity of the ecliptic $\varepsilon = 23.44^\circ$ — the tilt of Earth's axis relative to its orbital plane. This angle is why the seasons exist. Without it $\delta = 0$ always, the Astronomical Triangle collapses to its equinox form, and every day everywhere on Earth would be 12 hours of light. The Declination Triangle is why the Terran Calendar has meaning — it is the geometric source of seasonal variation.

$$\delta = -0.40928 \cdot \cos(2\pi(\text{DOY}+10)/365)$$

Triangle 2 feeds δ into Triangle 1. Everything the Terran Series measures flows from this chain: the 23.44° tilt of Earth produces δ , δ enters the Astronomical Triangle, and the Astronomical Triangle produces every instrument.

Triangle 3 — The Position Triangle (O₁-O₂-P)

The Position Triangle connects any two observers on Earth through the North Pole. It is the triangle of the Terran Position — the geometric relationship between two of the 4,344 solar identities the Series has mapped.

The three vertices:

- O₁ — Observer 1, at latitude φ_1 and longitude λ_1
- O₂ — Observer 2, at latitude φ_2 and longitude λ_2
- P — the North Pole

The three sides:

- PO₁ = $90^\circ - \varphi_1$ (co-latitude of observer 1)
- PO₂ = $90^\circ - \varphi_2$ (co-latitude of observer 2)
- O₁O₂ = great circle distance between the two observers

The angle at P = $\Delta\lambda$ = difference in longitude between the two observers.

Applying the spherical law of cosines:

$$\cos(d) = \sin(\varphi_1) \cdot \sin(\varphi_2) + \cos(\varphi_1) \cdot \cos(\varphi_2) \cdot \cos(\Delta\lambda)$$

This formula gives the great circle distance — the shortest path between any two points on Earth. Two observers at the same latitude but opposite hemispheres have a Position Triangle that reveals exactly how different their solar experience is. Triangle 3 is independent of the sun — it connects positions, not sun and observer.

Triangle 4 — The Terminator Triangle (T-E-P)

The terminator is the great circle dividing the illuminated half of Earth from the dark half at any moment. The Sphere Clock draws it in real time. The Terminator Triangle connects

a point on the terminator (T), the point where the terminator crosses the equator (E), and the North Pole (P).

The terminator is always perpendicular to the direction of the sun — it is the great circle whose pole is the sub-solar point. Its tilt relative to Earth's axis is exactly equal to the solar declination δ . At the equinoxes ($\delta = 0^\circ$) the terminator passes through both poles — every latitude on Earth receives exactly 12 hours of light. At the solstices ($\delta = \pm 23.44^\circ$) the terminator is most tilted — one pole is in perpetual light, the other in perpetual dark.

$$\phi_{\text{terminator}} = \arctan(-\cos(H) / \tan(\delta))$$

Triangle 4 is the visual signature of everything Triangle 1 calculates. Every line the Sphere Clock draws across the globe is a solution to the Terminator Triangle.

Triangle 5 — The Equinox Triangle (the perfect triangle)

At the equinoxes — March 20 and September 23 — the solar declination $\delta = 0$. The Astronomical Triangle becomes its most symmetric form: all three sides equal 90° .

- $ZP = 90^\circ - \phi$ (co-latitude)
- $PX = 90^\circ - 0^\circ = 90^\circ$ (polar distance at equinox)
- $ZX = 90^\circ$ (zenith distance at horizon, always)

When all three sides equal 90° , all three angles also equal 90° . The spherical excess is exactly 90° . This is the maximum possible symmetry in the Astronomical Triangle — and it occurs simultaneously at every latitude on Earth.

This is the geometric proof of why the Terran Calendar anchors at the equinox. At this moment — and only at this moment — the sphere treats every observer identically. The day is 12 hours everywhere. The triangle is the same everywhere. The sphere is maximally honest. The Terran New Year begins here because this is the one moment when the geometry gives every human being on Earth the same triangle.

Triangle 6 — The Solstice Triangle (the frozen triangle)

At the solstices — June 21 and December 21 — the solar declination reaches its maximum magnitude: $\delta = +23.44^\circ$ (NH summer) or $\delta = -23.44^\circ$ (NH winter). The Astronomical Triangle is at its most distorted.

At the solstice the rate of change of declination $d\delta/dt = 0$ — the sun's declination momentarily stops changing. This means the day length stops changing. The Kairos signal — dL/dd — drops to zero. Kinesis Pulse reaches its annual maximum duration (summer) or minimum (winter). Sol sistere — the Latin root of 'solstice' — means the sun stands still. The Solstice Triangle is the frozen triangle, the moment when the geometry holds its breath.

Above the Arctic or Antarctic Circle at their respective solstices, the Solstice Triangle has no solution — the sun never sets (polar day) or never rises (polar night). The triangle fails because the geometry of light has reached its physical limit.

Triangle 7 — The Solar Flux Triangle (Z-S-N)

The Flux Calculator requires the solar zenith angle — the angle between the sun and the zenith. The Solar Flux Triangle connects the Zenith (Z), the Sun (S), and the Nadir (N) — the point directly below the observer on the opposite side of Earth.

The angle at Z = the solar zenith angle (SZA). Solar altitude = $90^\circ - \text{SZA}$.

$$\cos(\text{SZA}) = \sin(\varphi) \cdot \sin(\delta) + \cos(\varphi) \cdot \cos(\delta) \cdot \cos(H)$$

Solar irradiance at the surface is proportional to $\cos(\text{SZA})$ — the projection of the sun's rays onto the horizontal surface. When the sun is directly overhead ($\text{SZA} = 0^\circ$) the full solar constant $S_0 = 1,361 \text{ W/m}^2$ is delivered. As the sun approaches the horizon ($\text{SZA} \rightarrow 90^\circ$) the energy per unit area diminishes toward zero. The 6.9% Southern Hemisphere perihelion advantage the Series documents flows directly from Triangle 7 — $\cos(\text{SZA})$ is larger when Earth is closer to the sun, and the Southern Hemisphere's summer coincides with perihelion.

Triangle 7 is Triangle 1's daylight companion. Where Triangle 1 operates at the horizon — calculating when light begins and ends — Triangle 7 operates throughout the day, calculating the quality of the light at every moment between sunrise and sunset.

The Geometric Family

Seven spherical triangles. Seven relationships. Seven instruments. One sphere.

- Triangle 2 produces δ → feeds into Triangle 1
- Triangle 1 produces H → feeds into Triangles 4 and 7
- Triangle 5 is Triangle 1 at $\delta = 0$ — the equinox, the calendar anchor
- Triangle 6 is Triangle 1 at $\delta = \pm 23.44^\circ$ — the solstice, the Kairos zero
- Triangle 3 stands alone — connecting positions, not sun and observer
- Triangle 7 completes the daylight picture — energy at every moment between rise and set

The Terran Series did not choose these triangles. It rediscovered them by asking one honest question: what does the sphere owe the people living on it? The answer came in seven triangles — a complete geometric family that was always there, waiting to be named.

The Extended Family — Five More Triangles

The seven triangles documented above are the biological layer — the triangles that connect observer, sun, and sphere. Mapping the complete Terran Series across all four temporal layers reveals five more spherical triangles, each belonging to a different layer and producing a different instrument.

T1 — The Meridian Triangle (O-P-G) — Temporal Layer

The Terran Meridian established that every degree of longitude represents four minutes of solar time. The geometry behind that statement is the Meridian Triangle — connecting the Observer (O), the North Pole (P), and the point where the observer's meridian meets the Greenwich Meridian (G) on the celestial sphere.

The three vertices:

- O — the observer, at longitude λ
- P — the North Pole
- G — the Greenwich Meridian reference point

The angle at P = λ , the observer's longitude. Each degree of that angle corresponds to 4 minutes of solar time offset from Greenwich. The Terran Day uses this triangle to correct sunrise and sunset from solar time to local clock time when the visitor enters their longitude.

$$\text{timezone offset} = \lambda / 15 \text{ hours}$$

Atlanta ($\lambda = -84.39^\circ$) sits 9.39° west of its timezone center (-75°), adding 37.6 minutes of solar offset. London ($\lambda = 0^\circ$) has no offset. Tokyo ($\lambda = 139.7^\circ$) sits 4.7° east of its center, subtracting 18.8 minutes. The Meridian Triangle is the geometry of why clocks and sun do not always agree.

T2 — The Solar Clock Circle (M-N-Z) — Temporal Layer

The Solar Clock (Universal Times, R. Gossett, 2026) treats the day as a circle — 360° of rotation divided into 10 loops, each loop into 100 arcs, each arc into 100 ticks. This is a temporal analog of spherical geometry: the day circle is a great circle of time, and the Solar Clock reads position on it the way latitude and longitude read position on the spatial sphere.

The three reference points:

- M — Solar Midnight (0° on the day circle, t:0:00:00)
- N — Solar Noon (180° on the day circle, t:5:00:00)

- Z — the current moment (any point on the circle)

The day circle geometry:

$$\mathbf{1\ loop = 36^\circ = 2.4\ hours \quad 1\ arc = 0.36^\circ = 86.4\ seconds \quad 1\ tick = 0.0036^\circ = 0.864\ seconds}$$

The Solar Clock is the temporal layer's answer to the Astronomical Triangle: where Triangle B1 measures position relative to the spatial sphere, Triangle T2 measures position relative to the temporal circle. Together they cover both dimensions of the Terran Series' four-dimensional framework.

B7 — The Kairos Triangle (D₁-D₂-P) — Biological Layer

The Kairos instrument measures dL/dd — the first derivative of day length, the rate at which the photoperiod is changing from one day to the next. This signal has a geometric source that the Series has not yet named: the Kairos Triangle.

The three vertices:

- D₁ — the sun's position on the celestial sphere today
- D₂ — the sun's position on the celestial sphere tomorrow
- P — the North Celestial Pole

The arc D₁-D₂ is the daily change in solar declination $d\delta/dt$. At the equinoxes, the sun moves along the ecliptic at its fastest rate relative to the equator — $d\delta/dt = 0.4036^\circ/\text{day}$ — and the Kairos Triangle has its maximum area. At the solstices, the sun's declination momentarily stops changing — $d\delta/dt \approx 0$ — and the Kairos Triangle collapses to zero area.

The area of the Kairos Triangle IS the Kairos signal — made geometric, visible, and precise. The equinox is the moment of maximum triangle area. The solstice is the moment of zero triangle area. Sol sistere — the triangle vanishes because the sun has stopped moving between declinations.

E1 — The Moment Triangle — Experiential Layer

The Terran Moment reads four dimensions simultaneously: Movement, Soul Month, Kinesis position, and Kairos signal. Each of these is encoded in the sides and angles of a single triangle — the Moment Triangle, which is the Astronomical Triangle (B1) interpreted experientially rather than computationally.

The three sides read as an experiential address:

- $ZP = 90^\circ - \varphi$ — your co-latitude, your Terran Position. The side that does not change. It is who you are on the sphere.
- $PX = 90^\circ - \delta$ — the polar distance of the sun today. It encodes the season and therefore the soul month. As δ changes across the year, this side changes — and the soul month changes with it.
- $ZX = 90^\circ$ — the zenith distance at the horizon. Always 90° . The fixed side — the horizon — the boundary between your light and your dark.

The angle H — the hour angle solved from this triangle — is the total arc of available light. The Kinesis reading $P:MM:PP$ is your current position within that arc, expressed as a decimal fraction. The Kairos signal is the rate at which the triangle's shape is changing. The Terran Moment portrait reads this triangle simultaneously at all its scales.

E2 — The Day Arc Lune — Experiential Layer

A spherical lune is the wedge between two great circles that meet at both poles — like a slice of an orange. The Day Arc Lune is the lune whose angle is $2H$ — twice the hour angle at sunrise and sunset — the total arc of today's light projected onto the celestial sphere.

The Day Arc Lune has two arcs:

- Light arc = $2H$ — the arc swept by the sun from sunrise to sunset
- Dark arc = $2\pi - 2H$ — the remaining arc of the night

The area of the lune is proportional to $2H$. At the equinox ($H = 90^\circ$) the lune covers exactly half the sphere — the light and dark arcs are equal. In summer ($H > 90^\circ$) the light lune is larger than the dark. In winter ($H < 90^\circ$) the dark lune is larger. The ratio of light lune

area to total sphere area is precisely the photoperiod fraction that Kinesis measures as P:MM:PP.

The Day Arc Lune is the shape of your day on the sphere. Kinesis is the instrument that reads your position within it. The Terran Day shows the complete lune — light arc in gold, dark arc in navy — as its 24-hour ring.

The Complete Family — Eleven Triangles

Including the Position Triangle (documented separately in the Terran Position seed document), the Terran Series rests on twelve spherical triangles across four temporal layers. Excluding position, there are eleven — two temporal, seven biological, and two experiential.

- Temporal: T1 (Meridian), T2 (Solar Clock Circle)
- Biological: B1 (Astronomical), B2 (Declination), B3 (Terminator), B4 (Solar Flux), B5 (Equinox), B6 (Solstice), B7 (Kairos)
- Experiential: E1 (Moment), E2 (Day Arc Lune)

The chain of derivations runs: $B2 \rightarrow B1 \rightarrow B3, B4, B7 \rightarrow E1, E2$. T1 corrects the temporal reading for longitude. T2 maps the corrected time onto the Solar Clock circle. Together the eleven triangles form the complete geometric architecture of the Terran Series — eleven views of the same sphere, each producing a different instrument, all asking the same question: what does the sphere owe the people living on it?

Part VII — The Spherical Triangles

The Astronomical Triangle — Where the Formula Comes From

Every instrument the Terran Series has built — the Latitude Clock, the Day Length Calendar, the Kairos, Kinesis, the Flux Calculator, the Terran Day — rests on a single formula: $\cos(H) = -\tan(\varphi) \times \tan(\delta)$. That formula is not an axiom. It is a conclusion —

the solution to a spherical triangle that has been drawn on the celestial sphere for as long as navigators have read the sky.

A spherical triangle is a triangle drawn on the surface of a sphere where each side is an arc of a great circle — the largest possible circle on the sphere's surface. Unlike flat triangles, the angles of a spherical triangle always sum to more than 180° . The excess beyond 180° is proportional to the area of the triangle on the sphere — a small triangle near the pole has less excess than a large triangle spanning the equator. This excess is not an error. It is the geometry of the sphere asserting itself.

The Three Vertices

The astronomical triangle that produces the Terran Series formula has three vertices:

- Z — the Zenith. The point directly above the observer on the celestial sphere. It moves with the observer's latitude.
- P — the North Celestial Pole. The point where Earth's rotation axis, extended outward, meets the celestial sphere. It sits at an angular height equal to the observer's latitude above the horizon.
- X — the Sun. At the precise moment of sunrise or sunset, the sun sits exactly on the horizon — 90° from the zenith.

The Three Sides

The three sides of the astronomical triangle are arcs connecting these vertices:

- ZP = co-latitude = $90^\circ - \varphi$. The arc from the zenith to the pole. At the equator ($\varphi = 0^\circ$), this arc is 90° . At the North Pole ($\varphi = 90^\circ$), it collapses to zero.
- PX = polar distance = $90^\circ - \delta$. The arc from the pole to the sun. δ is the solar declination — how far north or south of the celestial equator the sun sits on this day of the year. At the equinoxes, $\delta = 0^\circ$ and PX = 90° . At the solstices, $\delta = \pm 23.44^\circ$.
- ZX = zenith distance = 90° . The arc from the zenith to the sun at the horizon. This is always 90° at sunrise and sunset — the sun is always exactly one quarter of a great circle from the zenith when it sits on the horizon. This fixed side is what makes the formula solvable.

The Derivation

The spherical law of cosines states that for any spherical triangle with sides a , b , c and angles A , B , C where each angle is opposite its corresponding side:

$$\cos(a) = \cos(b) \cdot \cos(c) + \sin(b) \cdot \sin(c) \cdot \cos(A)$$

Applying this to the astronomical triangle, with:

- $a = ZX = 90^\circ$ (zenith distance at horizon)
- $b = PX = 90^\circ - \delta$ (polar distance)
- $c = ZP = 90^\circ - \varphi$ (co-latitude)
- $A = H$ (the hour angle at the pole — what we solve for)

The substitution proceeds:

$$\cos(90^\circ) = \cos(90^\circ - \delta) \cdot \cos(90^\circ - \varphi) + \sin(90^\circ - \delta) \cdot \sin(90^\circ - \varphi) \cdot \cos(H)$$

$$0 = \sin(\delta) \cdot \sin(\varphi) + \cos(\delta) \cdot \cos(\varphi) \cdot \cos(H)$$

$$\cos(H) = -\sin(\delta) \cdot \sin(\varphi) / (\cos(\delta) \cdot \cos(\varphi))$$

$$\cos(H) = -\tan(\varphi) \cdot \tan(\delta)$$

This is the formula the Terran Series uses everywhere. It is not borrowed convention — it is a direct consequence of the sphere's geometry, derived from first principles using nothing more than the law of cosines applied to three arcs on a sphere.

What the Spherical Triangle Reveals

The spherical triangle makes three things visible that the formula alone does not:

- The formula is exact — not an approximation. The sphere is the model. There is no flat-Earth simplification hiding inside $\cos(H) = -\tan(\varphi) \times \tan(\delta)$. It is derived from the sphere directly.
- The formula encodes both hemispheres simultaneously. The spherical triangle is symmetric — the same derivation works at -60°S as at $+60^\circ\text{N}$. The hemisphere asymmetry the Terran Series addresses is not a flaw in the mathematics. It is a flaw in the calendar that was built from it.

- The spherical excess is the geometric signature of latitude. At the equator the astronomical triangle's angles sum to 270° — an excess of 90° above the flat-triangle limit of 180° . At higher latitudes the triangle distorts, the excess changes, and the day length changes with it. The spherical excess IS the latitude effect the Terran Series was built to name.

The astronomical triangle — three vertices, three sides, one derivation — is the geometric foundation beneath every instrument the Terran Series has built. Kinesis breathes because of it. The Kairos measures its rate of change. The Terran Calendar is anchored to the equinox because that is where $\delta = 0$ and the triangle is most symmetric — the one moment in the year when the sphere treats both hemispheres identically.

VIII. Statement of Origin

The Terran Sphere is the fourteenth work of the Terran Series, authored by Lee-Kah Williams and first archived in August 2026. It is released under Creative Commons Attribution 4.0 International (CC BY 4.0). You are free to share and adapt this work for any purpose, provided appropriate credit is given.

The Terran Series is archived at terranseries.com and on Zenodo under permanent DOIs. The Series represents an independent philosophical, calendrical, and astronomical framework developed without institutional affiliation.

The mathematical calculations in this document — day length formula, flux calculations, ecliptic coordinates — are derived from standard spherical astronomy and are independently verifiable. The philosophical framework is original to the Terran Series.

Reference to Universal Times (Gossett, 2026) is made with respect for an independent body of work whose entropy framework provided valuable language for articulating the structural case for calendar reform. That reference does not imply endorsement or collaboration.

The Terran Soul · The Terran Soul Star · The Terran Calendar · The Dual Zodiac · The Dual Veda · The Dual Dragon · The Terran Series: An Introduction · The Terran Calendar of the Soul · The Terran Practice · The Terran Meridian · Common Questions · The Terran Infinity Clock · The Terran Chronograph · The Terran Sphere

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